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AGGREGATE FILTER

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The phase-only Aggregate Filter improves optical rotation-invariant pattern recognition by combining the Fourier plane detection filters of multiple targets into one detection filter. By choosing targets that have minimal overlap in the Fourier plane, there is littl inter-target competition and an efficient detection of all the targets, or the rotated image that are widely separated in rotation angle (20^{o}) results in larger detection correlation spikes than for slightly rotated targets (5-15^{o}).

Modeling is also provided to predict the performance of the Aggregate Filter with 2,3,4, or 5 targets. A single parameter is used as a measure of the overlap of the Fourier transforms of the chosen targets.

A proof is also given that the Aggregate Filter is the optimum phase-only system for multiple targets that are rotated versions of one image. The phase-only Aggregate Filter is formed by first adding the Fourier transforms of the chosen targets, second

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conjugating this sum, and third normalizing pixel by pixel to provide a filter with unit amplitude.

1.0 ABSTRACT

Optical pattern recognition is extremely effective for well defined targets, and naturally discriminates against small changes in the target shape. Attempts to reduce the discriminatory power of the Fourier plane filter to permit some adjustments such as size or perspective changes while still identifying the "overall shape" have been generally unsuccessful. These attempts usually involve reducing the information content in the detection filter through various forms of low pass filtering such as circular or radial smearing.

Our research is directed at increasing the information content to obtain optical pattern recognition of distorted or rotated or multiple targets. This is accomplished by combining multiple Fourier plane filters which are non-overlapping in the Fourier plane and therefore do not interfere with each other. This system fills the Fourier plane with useful information for detecting multiple targets. It is most effective when the targets are most dissimilar, therefore having the least overlap in the Fourier plane. This is opposite from past approaches aimed at smearing together several similar targets.

Our research confirms the prediction that combining multiple target detection filters is most effective for targets that are dissimilar. This is shown for the case of combining rotated targets where a decrease in effectiveness is noted for slightly rotated targets, followed by an increase in detection efficiency for the combining of detection filters from wildly rotated targets.

We also show that the optimal phase-only Aggregate Target Filter is obtained by making a phase only version of the sum of the various Fourier plane target filters. This optimum is from the point of view of an individual pixel and is not a global optimization.

Our research also shows the effectiveness of the Aggregate Filter in combining three targets, as well as four targets. The power in the output correlation spike decreases as expected as the number of target filters being combined increases. In one case for a single target filter defined as giving 100% power at the peak of the output correlation spike, the two target Aggregate Filter gave 82% power in the correlation spike, the three target Aggregate Filter gave 72% power and the four target Aggregate Filter gave 63% power in the correlation spike.

The two-target Aggregate Filter is shown to have the same sensitivity to slight rotations of the input scene as does a single target phase-only filter. The width of the correlation spike is also similar.

Our research also includes modeling of the efficiency of the Aggregate Filter as a function of a simple amplitude ratio parameter. Efficiency curves are given as a function of the most likely ratio of the Fourier transform amplitudes of the multiple targets at a typical pixel. These curves are presented for Aggregate Filters combining two to five targets, and provide an estimate of how well, for example, a five-target Aggregate Filter will perform based on experimental results from a two-target Aggregate Filter.

Our results are obtained by computer simulation of the optical correlator, and by modeling of the Aggregate Filter under the assumption of independent random phase in the Fourier plane.

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2.1 The Aggregate Filter Concept Developed at Aerodyne Research

The concept of the Aggregate Filter grows out of the phase only filter where the phase of pixels in the Fourier plane are chosen without any intention of being used by significant optical power from a chosen target. These pixels could be used to help detect another target. Thus the Aggregate Filter concept is to combine the Fourier plane filters of multiple dissimilar targets with the hope that there is little overlap. Under these circumstances a given pixel in the Aggregate Filter is utilized by only one target of the chosen set. This leads to the interesting statement that this multi-target optical detection system is optimum for targets that are very different in size or rotation angle, etc. Further this system will increase the useful information contained in the Fourier plane filter.

Applying the Aggregate Filter to rotation invariance is accomplished by combining greatly rotated versions of the target to minimize the interference. Several other similar Aggregate Filters will also then be necessary to fill in the gaps. This shows the Aggregate Filter concept to be completely different from other distortion invariant systems which handle incremental changes, and which therefore suffer from strong overlap in the Fourier plane.

3.0 AGGREGATE FILTER DEMONSTRATION

The Aggregate Filter concept proposes that it is more efficient to combine detection filters for several dissimilar targets rather than for several similar targets. A proper test of the basic concept must control the similarity of the targets. We chose rotated versions of a target to provide a smooth transition from being similar to being dissimilar.

The optical correlator with an Aggregate Filter was demonstrated by computer simulation. This primarily involves discrete Fourier transforms.

3.1 Highly Rotated Targets Show Increased Efficiency

Figure 1 shows the test target shape. It is designed to produce a Fourier spectrum with strong high frequency components in the y-direction, but which are relatively narrow in the x-direction, and to have a phase asymmetry.

The Aggregate Filter is formed by first adding the Fourier transform of the test target and the Fourier transform of a rotated version ($1°$-$85°$) of the test target. The Aggregate Filter is the Phase-Only version of the conjugate of that sum of transforms.

The Aggregate Filter is tested by multiplying it (array multiplication) by the Fourier transform of the input target, inverse Fourier transforming the product, and examining this array which is the correlation plane. The magnitude of the correlation spike is extracted, and is referred to in terms of its electric field throughout the body of this report.

Figure 2 shows the important result. The correlation spike amplitude, normalized to that of a perfect phase-only filter, is as shown obtained from a 2-target Aggregate Filter with one of the two targets as the tested input scene. The x-axis gives the rotation angle between the two targets used to produce the Aggregate Filter. The notable feature is the dip in detection efficiency for targets which are different enough to have incompatible phases

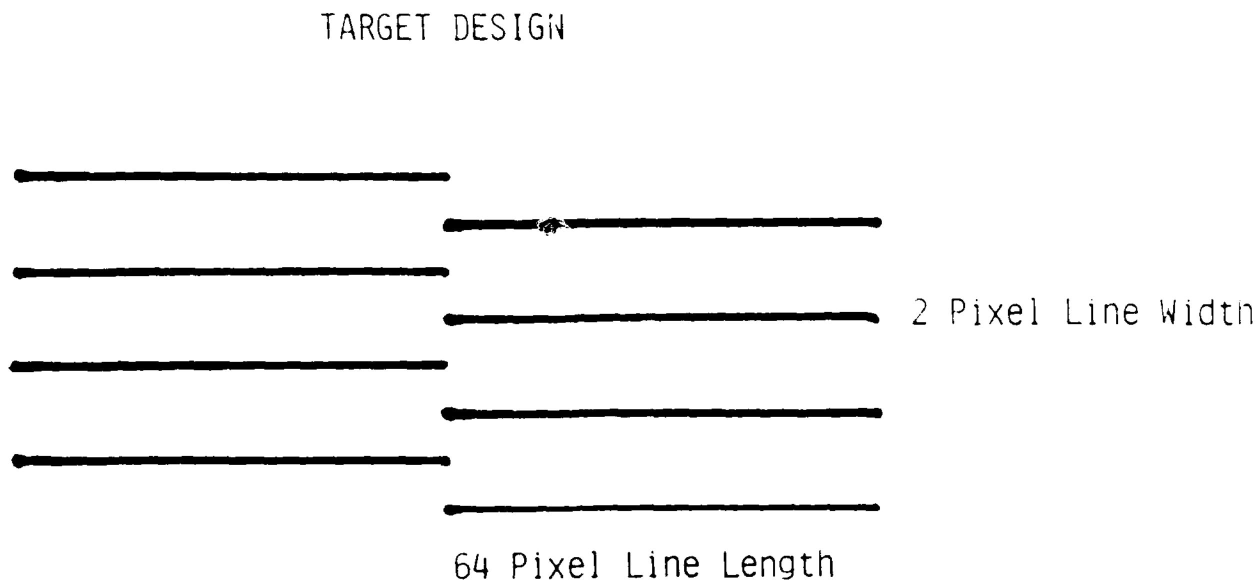

Figure 1. The target shape used throughout this report is shown here. It consists of a series of lines of length 64 pixels and width 2 pixels. The whole field of view is 256 x 256 pixels. This target shape produces strong high frequency components in the y-direction, though narrow in the X-direction.

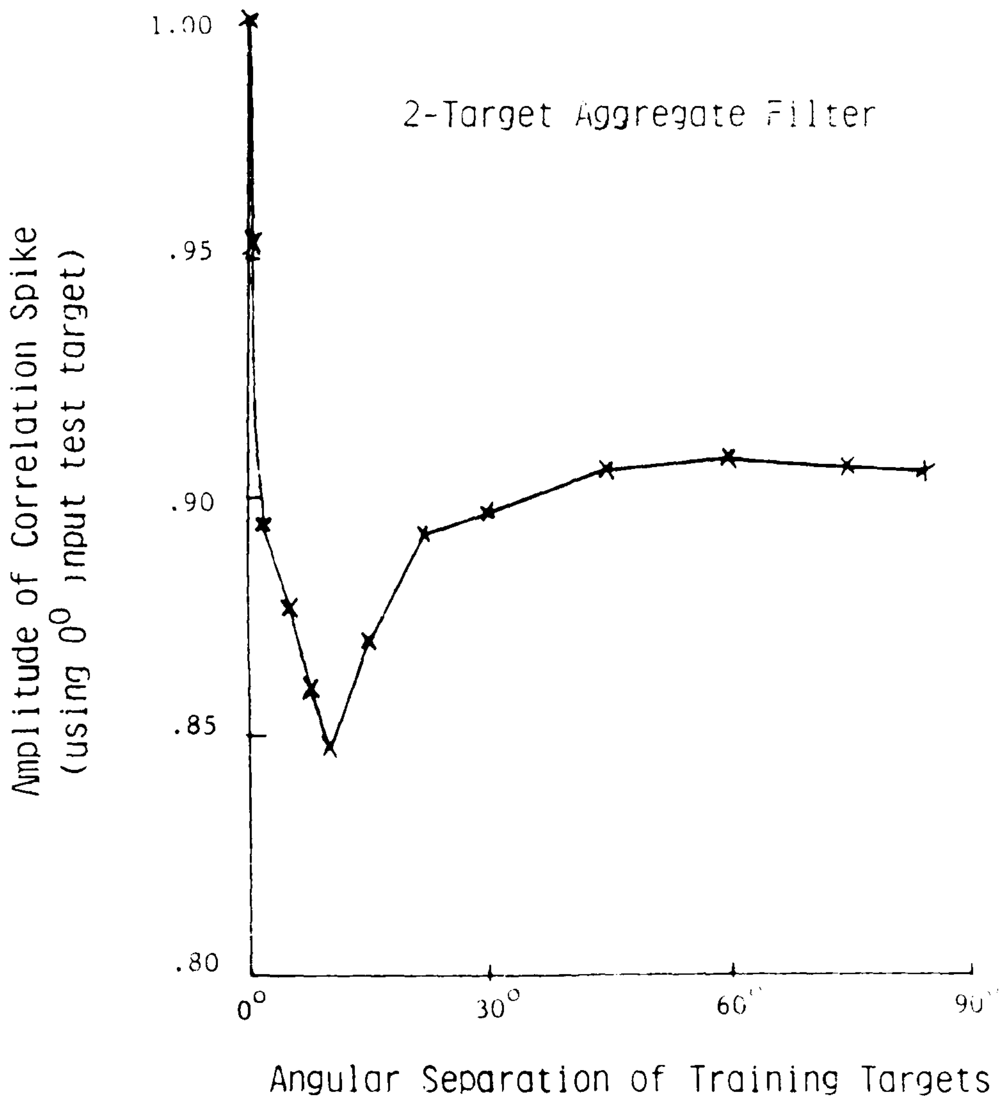

Figure 2. The 2-Target Aggregate Filters were designed for 0° and the angles shown on the X-axis. The input test target was at 0°. The dip in the amplitude of the correlation spike at low angles and subsequent recovery at large angles shows that the Aggregate Filter is more effective when designed for targets that are widely seperated in angle. This is due to the reduced overlap of the Fourier spectra of the chosen targets.

in their Fourier transforms but not different enough to be non-overlapping in their Fourier transforms. The point here is that the phase of a given pixel in the Aggregate Filter is dominated by the target with the largest amplitude at that Fourier plane pixel, and that there is little loss in efficiency to the other target which presents negligible power to that pixel during use.

The other features of interest in Figure 2 are the recovery and flattening of efficiency to over 90% after about 30°, and the rise to 100% efficiency near 0°. At 0° the two targets are alike and therefore 100% compatible. The flattening of the efficiency as opposed to a rise towards 100% with larger angles, is thought to be a result of competing effects. The low spatial frequency pixels of the two targets become incompatible only with large rotation angles and finally become nonoverlapping only with even larger rotations due to the small radii. Thus larger rotations progressively bring spectral components nearer the center of the Fourier plane into play. Further the Fourier transforms are frequently dense in the vicinity of zero frequency and therefore never achieve a non-overlapping condition from a rotation.

Three-Target and Four-Target Aggregate Filter Demonstration

Two Aggregate Filters were simulated, each using three targets. One with 0°, 10°, and 15° rotations as an example of targets which are too similar. The second filter was constructed from target rotations of 0°, 45°, and 75° as an example of dissimilar targets. The 0-10-15 filter gave a correlation peak (field) of 75% of the maximum possible, and the 0-45-75 filter gave 85%. The test input image was the 0° target. Here the affect of using dissimilar targets was even more noticeable.

Four-Target Aggregate Filters were also tested. The overly similar targets were 0°, 8°, 15°, and 22° and gave a correlation peak (field) of 71%. The dissimilar target set was 0°, 30° 60°, and 85° and gave 80%. Here again is evidence of the effectiveness of the Aggregate Filter concept, detecting four rotated versions of the same target with only a 20% loss in electric field strength at the correlation spike.

3.2 Sensitivity of the Aggregate Filter To Incremental Rotation of the Input Test Image

Thus far we have discussed the efficiency of the Aggregate Filter in detecting only targets for which it was specifically designed. Here we compare the sensitivity of a 2-Target Aggregate Filter to slight rotations of the input image.

Figure 3 shows the amplitude of the individually normalized correlation spikes from a straightforward phase-only filter, and from an Aggregate Filter designed for 0° and 45°, as a function of the rotation of the input test image in the vicinity of zero degrees. They are extremely similar, and quite sensitive to angle. This is expected since the sharp rotation sensitivity is due to the strong high spatial frequency components, and these are expected to be nonoverlapping in the Aggregate Filter.

Figure 4 shows a rough estimate of half-power width of the correlation spike for the simple phase-only filter and for the 0-45 Aggregate Filter, as a function of rotation of the input test image. The "width" of the correlation spike is calculated from the number of pixels (N) exceeding 70% (half power) of the peak.

$$\text{"width"} = \frac{\sqrt{N}}{256} \times 100\% \qquad \text{i.e. percent of correlation plane width}$$

This formula will give the correct half-power width if the spike is round and has no sub-peaks. The formula underestimates the width under other circumstances, and is used here to remove human bias. Figure 4 shows a similar width for the Aggregate Filters correlation spike for moderate rotation angles. Note that beyond 25° the amplitude has dropped about 80%. It is also noted that the apparent correlation spike width for the Aggregate recovered at 22° which happens to be about midway to the second target angle of 45°. However the correlation spike amplitude at 22° still remains low.

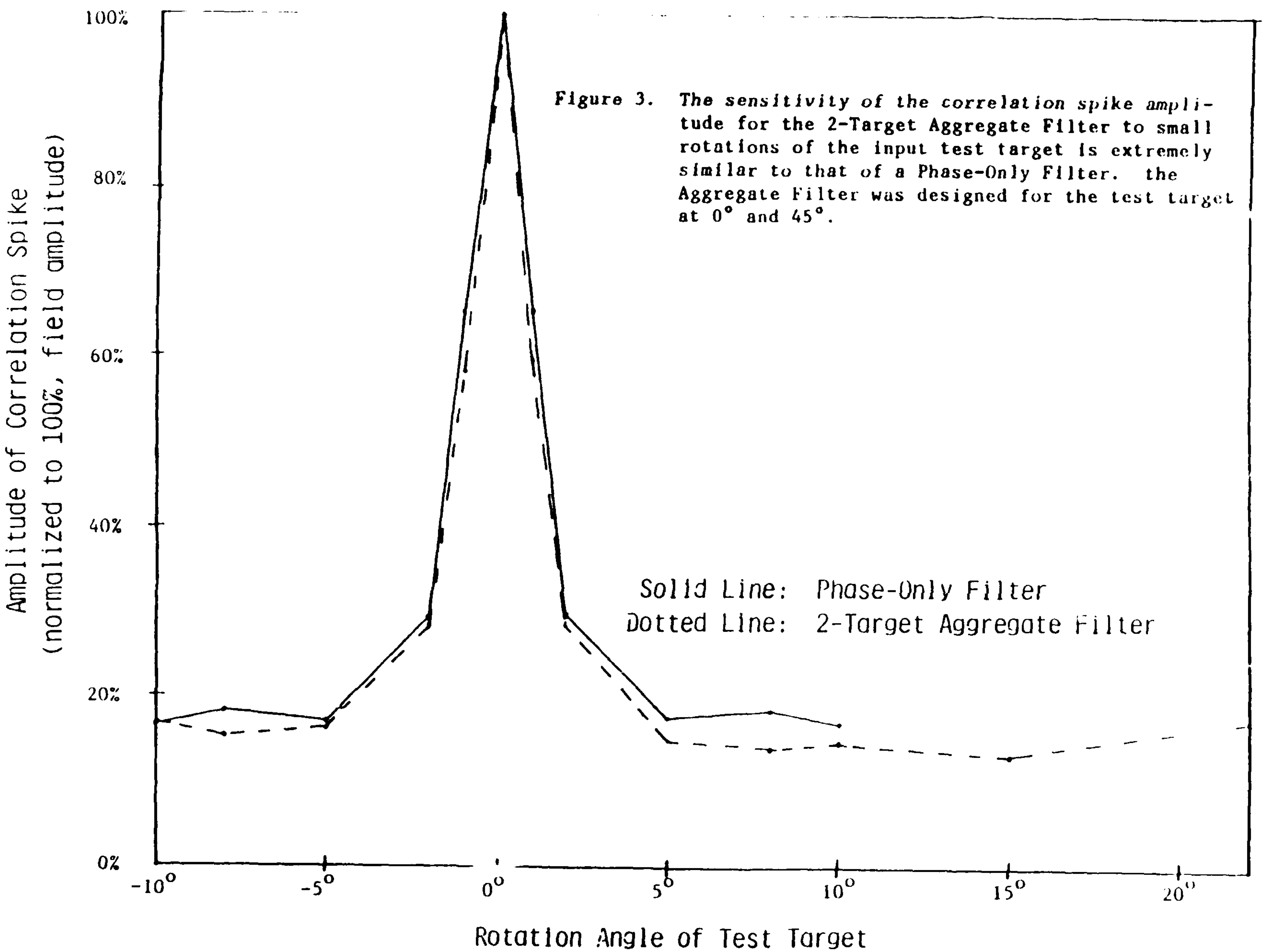

Figure 3. The sensitivity of the correlation spike amplitude for the 2-Target Aggregate Filter to small rotations of the input test target is extremely similar to that of a Phase-Only Filter. the Aggregate Filter was designed for the test target at 0° and 45°.

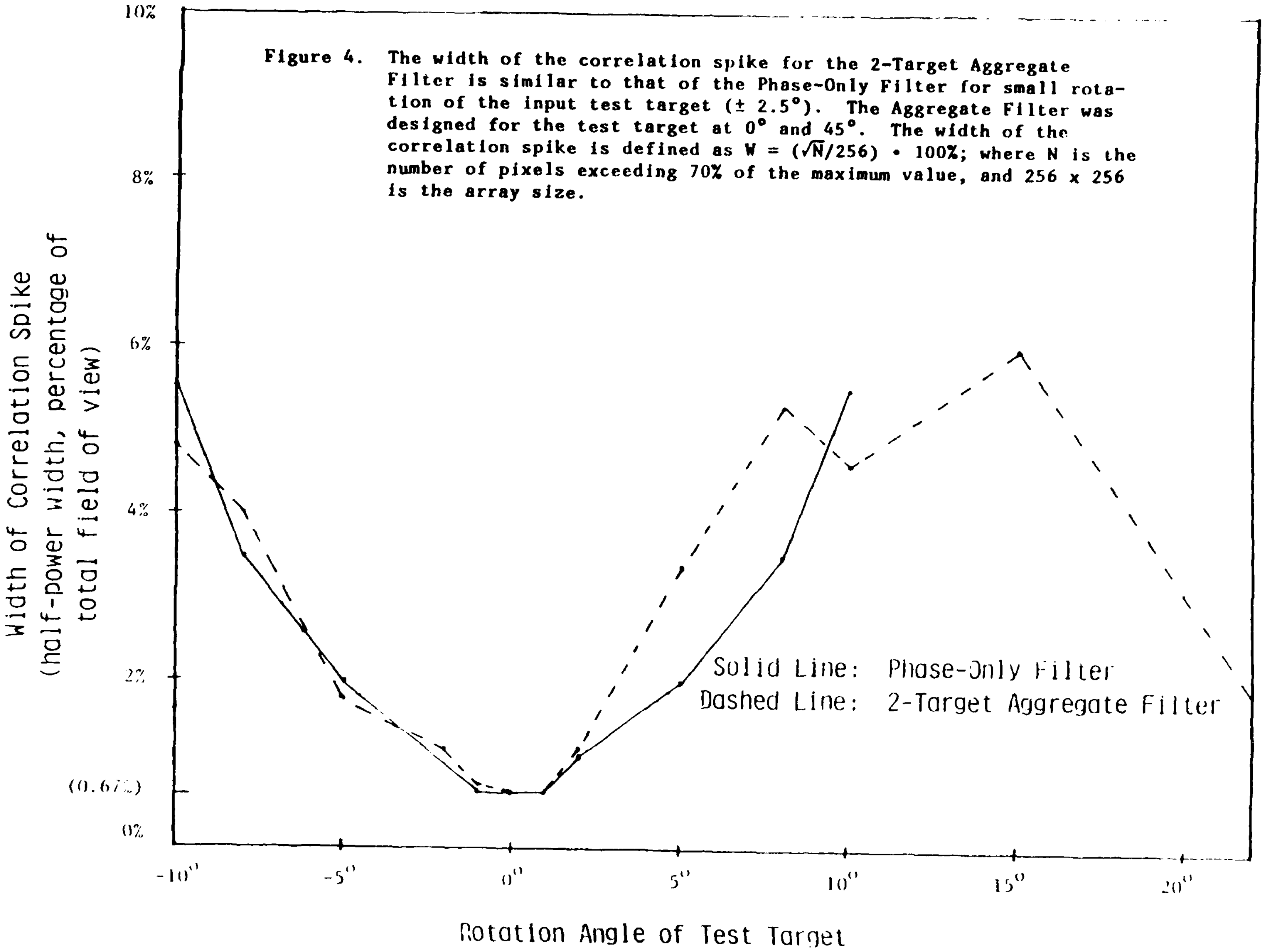

Figure 4. The width of the correlation spike for the 2-Target Aggregate Filter is similar to that of the Phase-Only Filter for small rotation of the input test target (± 2.5°). The Aggregate Filter was designed for the test target at 0° and 45°. The width of the correlation spike is defined as $W = (\sqrt{N}/256) \cdot 100\%$; where N is the number of pixels exceeding 70% of the maximum value, and 256 x 256 is the array size.

The rotation sensitivity of the correlation spike amplitude and width as produced by the Aggregate Filter has been shown to be quite similar to that of the straightforward phase only filter. The example used here was of an Aggregate Filter where the high frequency spectra of the targets were nonoverlapping.

4.0 OPTIMALITY OF THE AGGREGATE FILTER

Our Aggregate Filter is formed by taking the phase-only conjugate of the sum of the Fourier transforms of the chosen targets. One can conceive of other formulas for generating similar types of Aggregate Filters. For example, this research was begun with the intention of choosing the phase at each pixel to be equal to the phase of the target filter with the largest amplitude at that pixel. This latter design is based on the notion that there will be negligible overlap in the filters. However there is overlap, especially in the low frequency areas of the Fourier plane. The chosen phase is a compromise for the multiple targets, and the (vector) sum of the Fourier transforms is shown to be a local optimum.

Understanding this calculation involves knowledge of certain details about the optical correlator operation, as follows. Each pixel in the Fourier plane filter contributes an electric field to the correlation plane whose amplitude and spatial phase distribution do not depend on what fields are contributed by the other pixels. The field delivered by this specific filter pixel depends only on the field arriving at that location from the input image, and on the phase and attenuation of that filter pixel. The correlation spike is the simple vector sum of the contributed fields from all the pixels of the Fourier plane filter.

The facts related above permit a relatively simple calculation to determine the optimum phase for any specific filter pixel which will cause it to contribute the maximum field to the correlation spike. That is, the perturbing effect on the correlation spike due to a single pixel can be calculated and optimized. This is only a local optimum, not necessarily a global optimum. A global optimum would involve "simultaneously" adjusting the phases of all the pixels to get the absolute best performance for all the chosen targets. The global optimum is a much more difficult problem and could be attempted using a mathematical annealing process on an array of smaller size than the 256 x 256 array which was used in our simulations.

4.1 Proof

Assume a phase-only aggregate filter for multiple targets.

Assume the amplitude of the correlation spike for the n^{th} target is S_n.

The contribution of one specific pixel to the n^{th} correlation spike gives:

$$S_n + \varepsilon_n \left[\cos(\Delta\theta_n) + j\sin(\Delta\theta_n)\right]$$

where ε_n is the field delivered to the pixel by the n^{th} target (Fourier transform amplitude), and $(\Delta\theta_n)$ is the phase error between what is ideal for the n^{th} target and what is chosen as a compromise for all the targets. Note that the ideal phase is flat across the output plane of the Fourier plane filter as obtained by using the conjugate of the target's Fourier transform.

Optimize the sum of the power of the correlation spikes for all the targets:

$$\text{i.e. maximize } \sum_n \left| S_n + \varepsilon_n \left[\cos(\Delta\theta_n) + j\sin(\Delta\theta_n)\right]\right|^2$$

$$= \sum_n \left\{S_n^2 + 2\, S_n\, \varepsilon_n \cos(\Delta\theta_n) + \varepsilon_n^2 \cos^2(\Delta\theta_n) + \varepsilon_n^2 \sin^2(\Delta\theta_n)\right\}$$

$$= \sum_n \left\{S_n^2 + \varepsilon_n^2 + 2\, S_n\, \varepsilon_n \cos(\Delta\theta_n)\right\}$$

let $\Delta\theta_n = \theta_n - \theta_{final}$ where θ_n is the ideal phase for the n^{th} target at the chosen pixel, and θ_f is the compromise phase.

The optimum phase-only Aggregate Filter comes from the linear sum of the complex Fourier amplitudes of the targets, and it yields the least phase error for that target which has the overwhelmingly stronger amplitude, as was intended in the original concept.

5.0 MODELING THE AGGREGATE FILTER

We examine the efficiency of Aggregate Filters by calculating the effectiveness of the Fourier plane pixels in correcting the phase of their incident electric fields such that the correlation spike is as large as possible. The field efficiency is defined as $\cos(\Delta\theta)$ where $\Delta\theta$ is the angle between the ideal phase and the chosen phase of the pixel. In fact the actual field transferred to the correlation spike is more accurately described by:

$$\cos(\Delta\theta) + j\sin(\Delta\theta)$$

but the $\sin(\Delta\theta)$ will cancel on the average over many pixels. That is, for every pixel with a phase error $\Delta\theta$ there is on the average another pixel with a phase error $-\Delta\theta$ with a similar amplitude.

The term efficiency here refers to electric field, and the power in the correlation spike is proportional to the square of the field efficiency. 100% field efficiency means that all the filter pixels have exactly the ideal phase for generating the largest correlation spike possible.

The modeling does not depend on the array size. The phase at a given pixel for the various target Fourier transforms is assumed to be random and independent. The calculations involve multidimensional integrals over all possible phases of the various targets' Fourier transforms, and are determined by numerical integration.

5.1 Efficiency Curves for Aggregate Filters with Two to Five Targets

Two-Target Aggregate Filter

Consider one kind of worst case where the amplitudes of the Fourier transforms of the two targets are the same at each pixel, but the phases are random and independent of each other. The worst case phase occurs when target 1 needs 0° and target 2 needs 180° at the same pixel. The Aggregate Filter uses a vector sum which (in the limit in this case) is 90°. Both targets suffer 90° errors here and receive on average a zero contribution ($\cos 90^\circ = 0$). Targets requiring 0° and 90° with equal amplitude produce a vector sum giving 45° and a field efficiency of $\cos 45^\circ = .707$, which is more typical.

For equal Fourier amplitudes the vector sum is always midway between the two angles making the average efficiency very simple to calculate:

$$\text{Efficiency} = \frac{1}{\pi} \int_{0}^{\pi} \cos \frac{\phi}{2}\, d\phi = \frac{2}{\pi} = 0.637$$

Thus an Aggregate Filter made for two targets where there is total overlap of their Fourier spectra with equal amplitudes and random phases will have an efficiency of 63.7%. (A perfect phase-only filter for one target would produce a correlation spike of 100% [field]).

The advantages of the Aggregate Filter appear when the Fourier spectra of the two targets do not overlap. We model the degree of overlap as an amplitude ratio r. That is, the Fourier transform amplitude of one target or the other is larger by a factor of r. We calculated the overall efficiency by numerical integration over all possible phases of the two targets with a fixed amplitude ratio as shown in Figure 5. Note that this calculation involves the target of interest being represented by amplitude ratios of r and 1/r since the other target must be superior at an equal number of pixels. Further these calculated efficiencies are weighted by the actual field impinging on the pixel (choices of incident fields are r, 1).

Figure 5 shows three curves. The uppermost is the efficiency of the pixel for the target with the larger amplitude at that pixel. The lower curve is the average efficiency for the target with the lower amplitude. The middle curve, the composite efficiency is the important result, and includes the proper weighting, as follows:

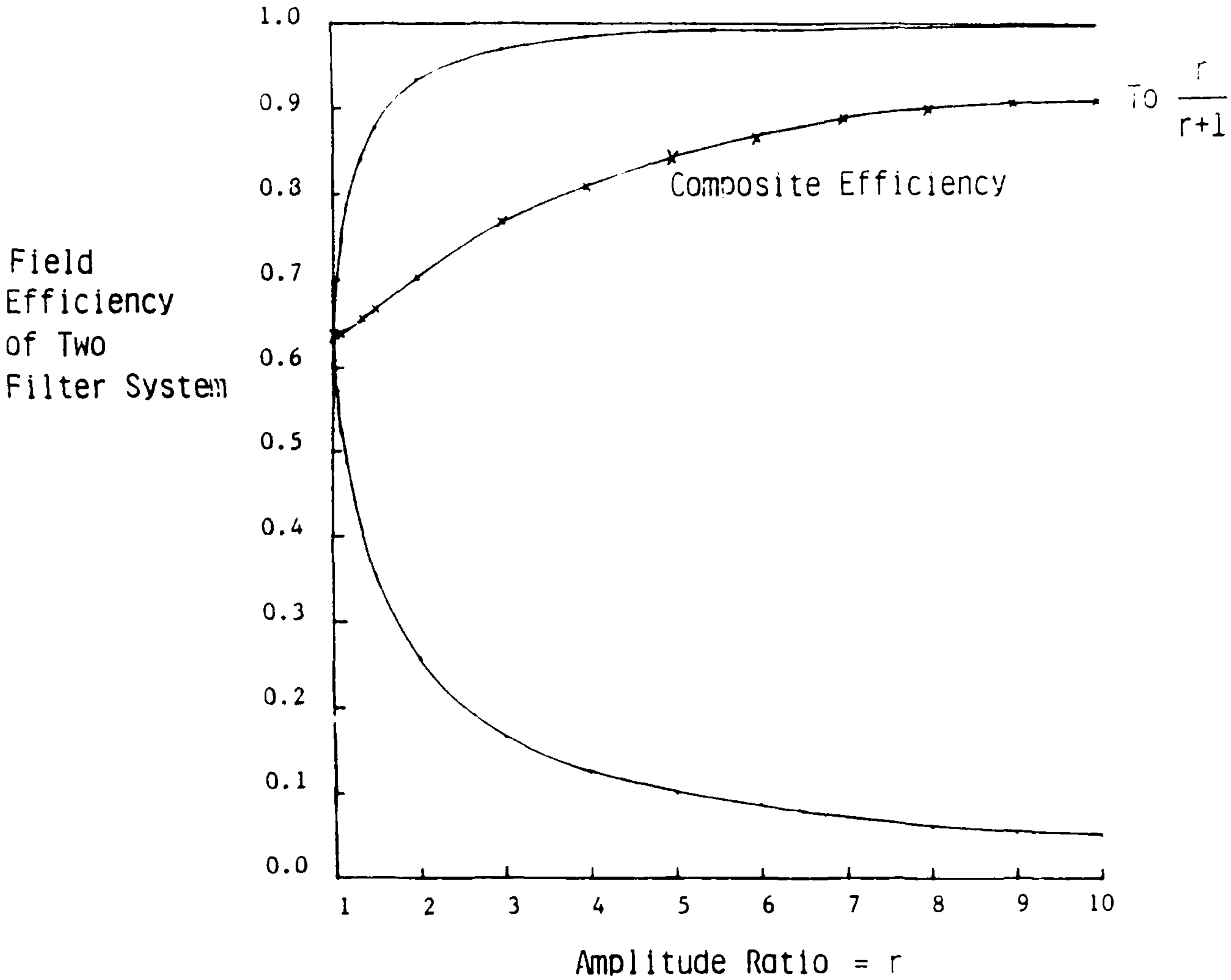

Figure 5. The Composite Efficiency curve gives the correlation spike amplitude (field) for an Aggregate Filter designed fcr two targets where the Fourier plane amplitude of one target is r times that of the other target at every pixel. The phases of the target Fourier spectra are assumed to be random and independent. An efficiency of 1.0 is obtained by a perfect phase-only filter designed for one target. The graph shows the increase in efficiency with an increasing value of r, which represents the degree non-overlap of the Fourier spectra of the two targets.

The uppermost curve is the average efficiency of a pixel acting for the target with the larger Fourier amplitude. The lower curve is the average efficiency for pixel acting for the target with the smaller Fourier amplitude. The composite efficiency is the combined upper and lower efficiency curves weighted by the involved fields, and normalized by the total fields of the two targets at that pixel. The composite efficiency gives the expected height of the correlation spike for the two targets compared to a perfect phase-only filter, as a function of the overlap parameter r.

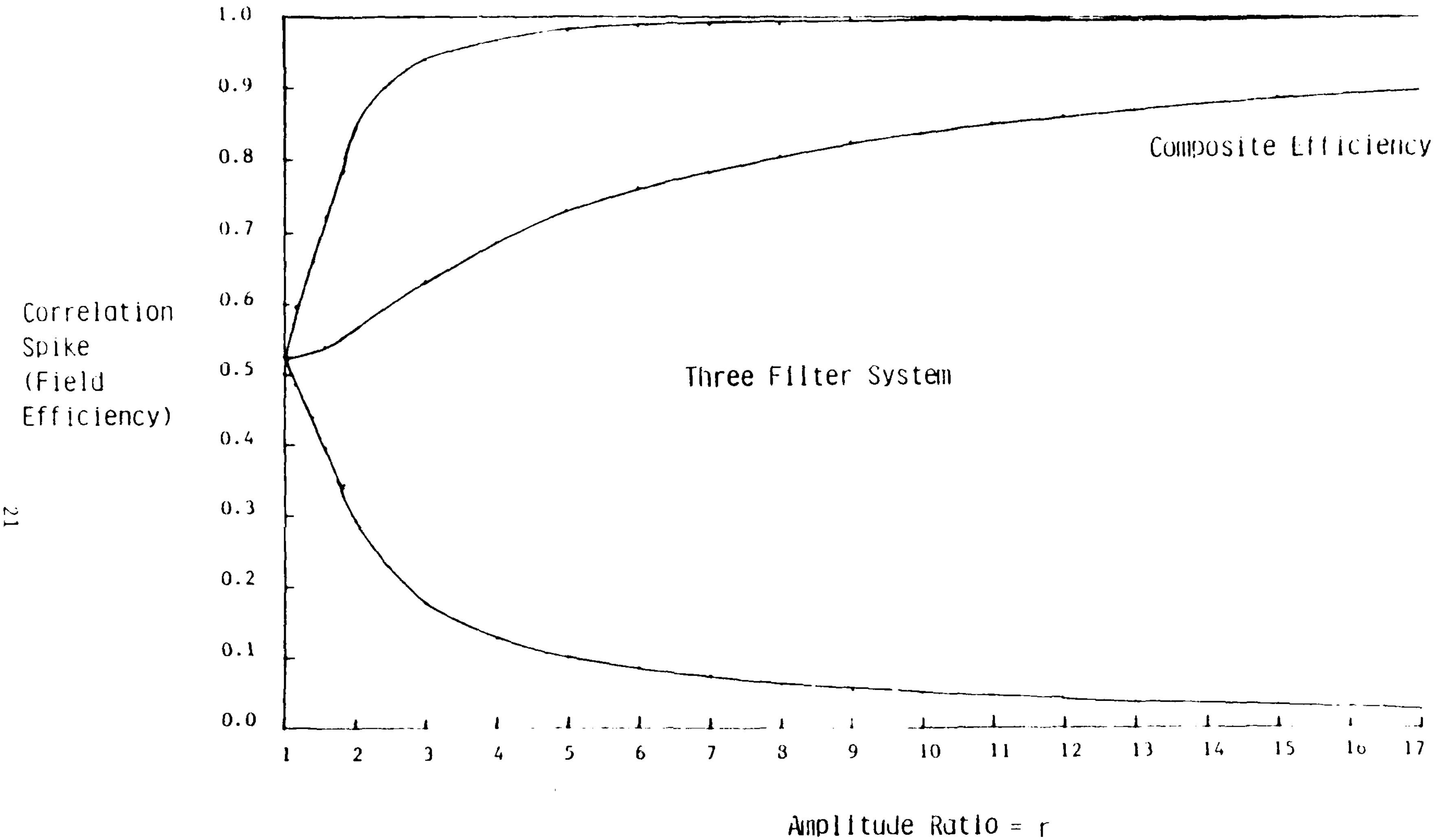

Figure 6. The Three-Target Aggregate Filter is modeled as a function of the overlap parameter r. The Composite Efficiency curve gives the expected amplitude of the correlation spike for the three targets compared to an ideal phase-only filter designed for one target.

The overlap parameter r is the ratio of the Fourier amplitudes from the three targets at a typical pixel. Two of the targets have an amplitude of 1 and the third target has an amplitude of r at the same pixel. Thus each target has the largest amplitude at one out of three pixels.

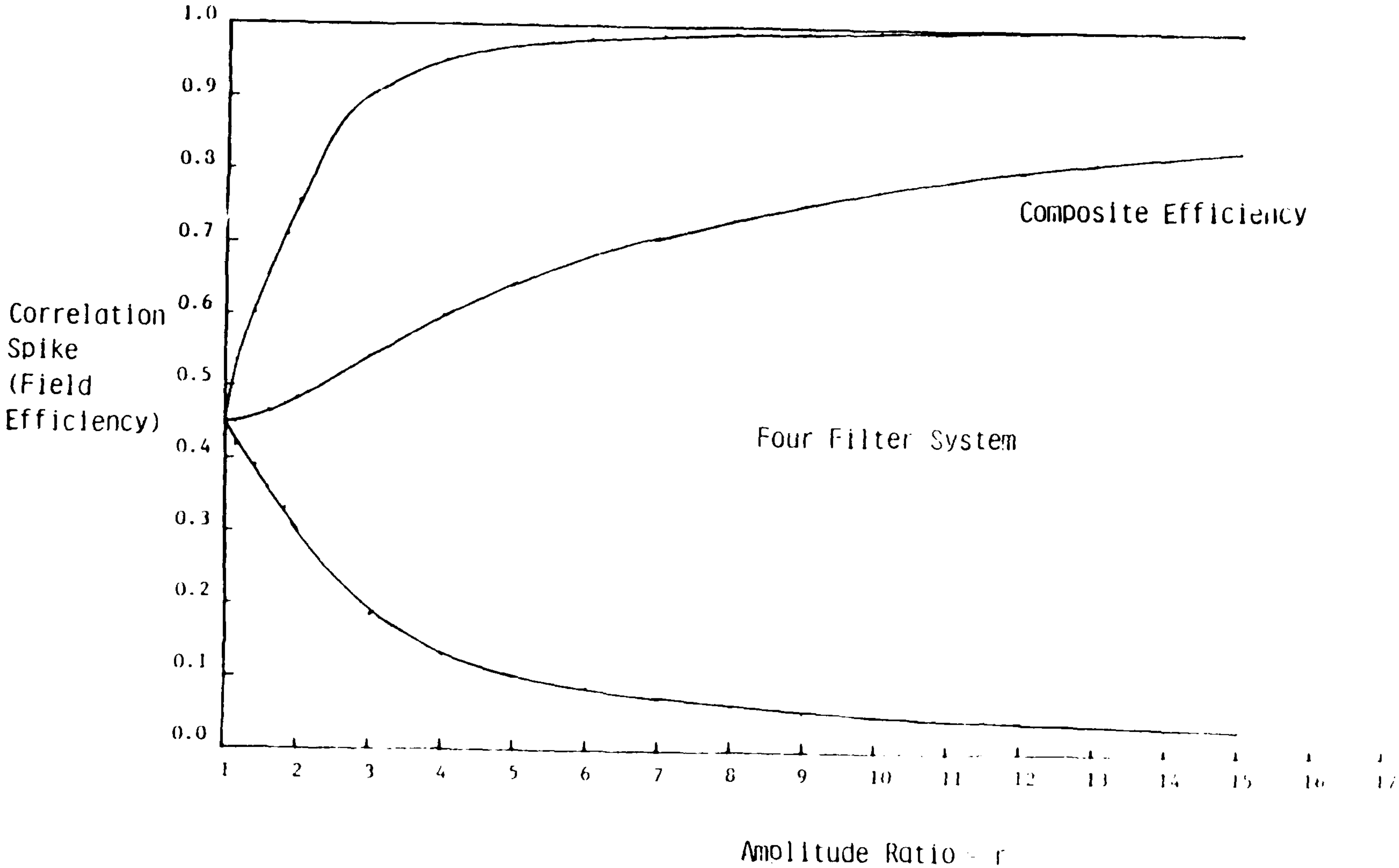

Figure 7. The Four-Target Aggregate Filter is modeled as a function of the overlap parameter r. The Composite Efficiency curve gives the expected amplitude of the correlation spike for the four targets compared to an ideal phase-only filter designed for one target.

The overlap parameter r is the ratio of the Fourier amplitudes from the four targets at a typical pixel. Three of the targets have an amplitude of 1 and the fourth target has an amplitude of r at the same pixel. Thus each target has the largest amplitude at one out of four pixels.

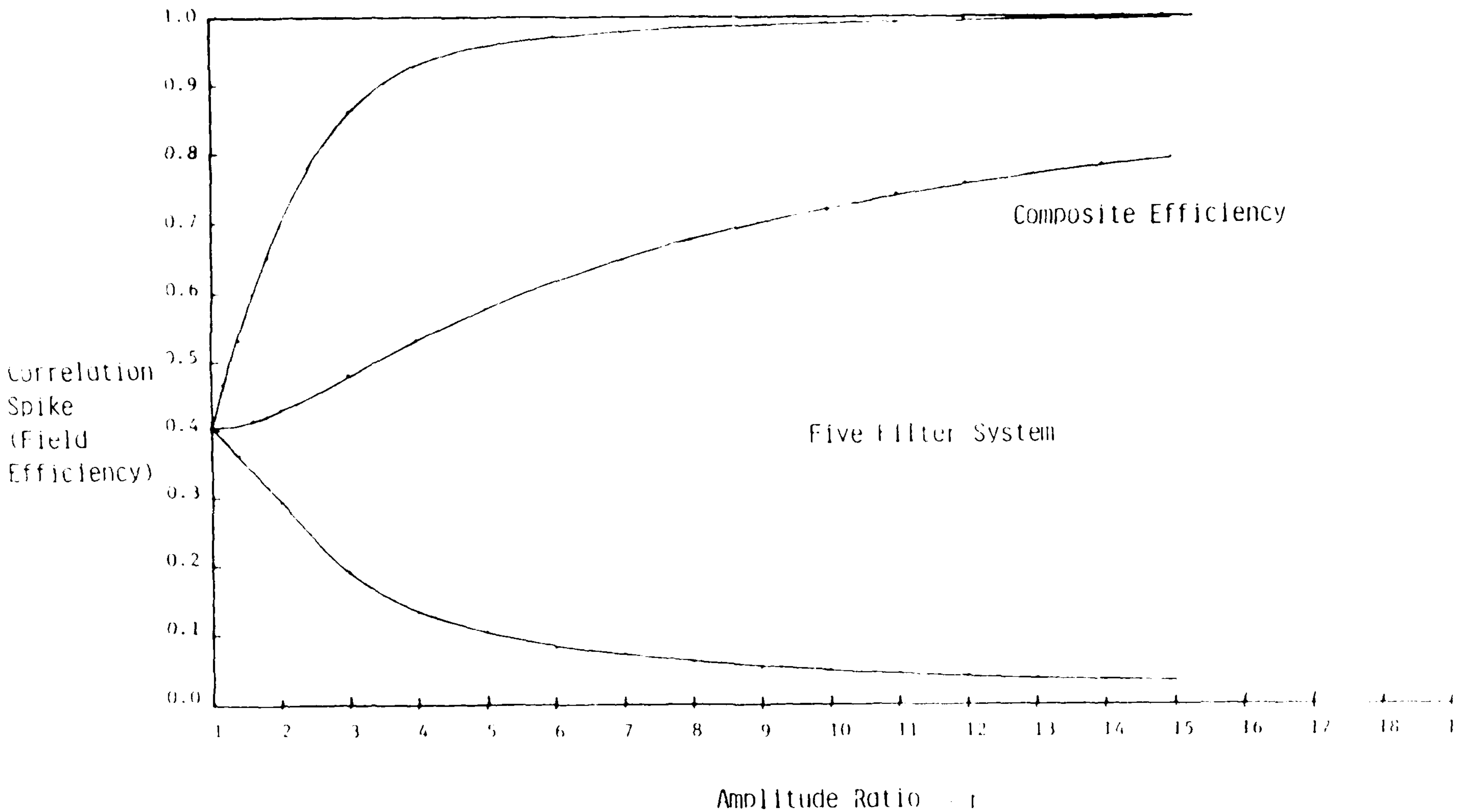

Figure 8. The Five-Target Aggregate Filter is modeled as a function of the overlap parameter r. The Composite Efficiency curve gives the expected amplitude of the correlation spike for the five targets compared to an ideal phase-only filter designed for one target.

The overlap parameter r is the ratio of the Fourier amplitudes from the five targets at a typical pixel. Four of the targets have an amplitude of 1 and the fifth target has an amplitude of r at the same pixel. Thus each target has the largest amplitude at one out of five pixels.

$$\text{Composite Efficiency} = \frac{r \cdot E_u + E_L}{r + 1}$$

E_u: higher efficiency with larger field of value r

E_L: lower efficiency with smaller field of value 1

For large values of r the efficiency increases, and the field wasted with the wrong phase is minimized, yielding an asymptotic limit of r/[r + 1] for the composite efficiency. The efficiency at r = 1 is 0.637 and checks with the value previously calculated.

The composite efficiency is the fraction of the electric field that could add to the correlation spike that in fact does contribute to the correlation spike with the proper phase. The composite efficiency is an average for one target over may pixels, some of which are individually efficient (have a near ideal phase) and some of which are dominated by another target. The formula weights the higher efficiency (E_u) with the larger field (r), and weights the lower efficiency (E_L) with smaller field (1). This weighted sum must be renormalized by dividing by r+1.

Three-Target Aggregate Filter

This model is quite similar to the two-target model above, and is a 2-D numerical integral over all possible phases of the three targets. Here we assume two targets with Fourier amplitudes of 1 and a third target with a Fourier amplitude of r at the typical pixel. Figure 6 shows the upper and lower efficiencies, and the composite efficiency:

$$\text{Composite Efficiency} = \frac{r \cdot E_u + 2 \cdot E_L}{r + 2} \quad \text{(weighted average)}$$

This formula indicates that for every pixel that yields high efficiency (E_u) for a specific target, there are two pixels which yield only the low efficiency (E_L) for that same target. For large amplitude ratios the composite efficiency is asymptotic to r/[r + 2]. The efficiency at r = 1 is 52.5%.

Four-Target Aggregate Filter

This model is similar to above and yields a composite efficiency beginning at 45.0% with r = 1 and asymptotically approaching r/[r + 3] as shown in Figure 7. It involves a 3-D numerical integration over all possible phases of the four targets.

$$\text{Composite Efficiency} = \frac{r \cdot E_u + 3 \cdot E_L}{r + 3}$$

Five-Target Aggregate Filter

This involves a similar four dimensional numerical integral and shows a composite efficiency beginning at 40.2% for r = 1 and approaching r/[r + 4] for large r as shown in Figure 8.

$$\text{Composite Efficiency} = \frac{r \cdot E_u + 4 \cdot E_L}{r + 4}$$

5.2 Using the Efficiency Curves to Predict Aggregate Filter Performance With Two to Five Targets

The efficiency versus amplitude-ratio curves provide a single parameter description of an Aggregate Filter. A single parameter is incomplete but it is an effective first step. Following are several examples of how to use the efficiency curves with comparisons to real data.

Case #1

Using rotated versions of the target shape shown in Figure 1 we experimentally (computer simulation) made two Aggregate Filters, each containing two targets, and measured their efficiencies (field) as:

Step 1 - 2-Target Aggregate Filter #1 (0°, 10°) = 84.8%

2-Target Aggregate Filter #2 (0°, 15°) = 87%

Average Efficiency = 85.9%

Step 2 - Check 2-target prediction curve at 85.9% (0.859 in Figure 5).

The implied value of r = 5.5

Step 3 - Predict the efficiency of a 3-Target Aggregate Filter using the targets at 0°, 10°, and 15° assuming r = 5.5 on the 3-target prediction curve (Figure 6).

r = 5.5 implies the 3-target efficiency will be 75%

Step 4 - Check experimental data for a 3-Target Aggregate Filter using 0°, 10°, and 15°.

Experimentally determined correlation was 75% (field) which is an excellent match with the prediction in Step 3. (The experimental test used a 0° target on the 0°, 10°, 15° filter).

Case #2

a) Experimentally determined efficiencies for:

2-Target Aggregate Filter (0°, 45°) = 0.906
2-Target Aggregate Filter (0°, 75°) = 0.906

Average Efficiency = 0.906

b) Check on 2-Target Prediction Curve for 90.6% (0.906 in Figure 5)

90.6% efficiency implies R = 9.5

c) Predict the efficiency of a 3-Target Aggregate Filter 0°, 45°, and 75° assuming r = 9.5 (Figure 6)

R = 9.5 implies the 3-Target Filter Efficiency will be 83%

d) Check Experimental Data for 3-Target Aggregate Filter using 0°, 45°, and 75° (tested with 0° target).

Experimental Correlation Peak = 84.7% which is a reasonable match with prediction in c).

Case #3

a) Real Data on 2-Target Aggregate Filters (16 pairs)

0°, 30°	0°, 60°	0°, 85°	Average Efficiency = 90.37%
0°, 10°	0°, 22°	0°, 30°	Average Efficiency = 87.90%
0°, 8°	0°, 15°	0°, 22°	Average Efficiency = 87.43%
0°, 8°	0°, 10°	0°, 15°	Average Efficiency = 85.90%

b) Get Implied Amplitude Ratios from 2-Target Curves (Figure 5)

90.37% implies R = 9.9
87.90% implies R = 7.0
87.43% implies R = 6.6
85.90% implies R = 5.5

c) Predict 4-Target efficiency from curves (Figure 7) for:

0, 30, 60, 85 implies efficiency = 75.9%
0, 10, 22, 30 implies efficiency = 71.1%
0, 8, 15, 22 implies efficiency = 69.9%
0, 8, 10, 15 implies efficiency = 66.2%

d) Compare with Experimental Data for 4-Target Aggregate Filter (tested with 0° target).

0, 30, 60, 85 yields an Experimental Efficiency = 79.6%
0, 10, 22, 30 yields an Experimental Efficiency = 72.7%
0, 8, 15, 22 yields an Experimental Efficiency = 70.6%
0, 8, 10, 15 yields an Experimental Efficiency = 70.4%

5.3 Modeling the Aggregate Filter With Random Fourier Plane Amplitudes

The single amplitude ratio parameter r is an oversimplification, but it is also an appropriate first step. An improvement might embody a typical amplitude distribution for target Fourier transforms. As a first step the efficiency of two- through five-target Aggregate Filters were calculated using random amplitudes for the Fourier transforms at a typical pixel.

The following efficiency values were calculated based on each Target's Fourier plane amplitude being randomly distributed (uniform distribution [0-1]). The phases are independent and random.

	Equal Amplitudes (r = 1)	Random Amplitudes	
2-Target	63.7%	73%	
3-Target	52.5%	59%	(Field Efficiency)
4-Target	45.0%	51%	
5-Target	40.2%	46%	

The increases for random amplitudes are 12 to 15% over the cases with equal amplitudes.

6.0 CONCLUSIONS AND RECOMMENDATIONS

We have demonstrated the Aggregate Filter concept which increases the information content in the Fourier Plane Filter to obtain rotation invariance or multiple target detection. This is accomplished by combining multiple target detection filters which have minimum overlap in the Fourier Plane. This was demonstrated by computer simulation using rotated versions of an image as the multiple targets and showed greater effectiveness when the targets were widely separated in rotation angle.

The Aggregate Filter takes a wholly new direction in distortion invariant pattern recognition research which was previously dominated by reduced information content filters. The primary problem with past work has been the strong overlap in the Fourier plane of the multiple targets which were intentionally chosen to be only incrementally different.

Examples of Aggregate Filters for 2, 3, and 4 targets were demonstrated. The sensitivity of 2-Target Aggregate Filters to incremental rotations of an input image was also shown to closely track the behavior of a similar phase only filter for a single target.

The Aggregate Filter was modeled with two through five targets using a single parameter called the amplitude ratio. This parameter was then used to predict the efficiency of Aggregate Filters with three and four targets based only on real data from 2-Target Aggregate Filters.

A proof was also presented which shows that the optimum phase-only Aggregate Filter is formed by complex addition of the Fourier transforms of the chosen targets, followed by extraction of the conjugate phase.

Finally modeling was begun on the effect that random amplitudes in the target Fourier transforms have on an Aggregate Filter. An expected increase in efficiency did appear with random amplitudes.

Recommendations

The Aggregate filter concept has been demonstrated and shown to be an optimal phase-only filter for multiple targets. Further research is needed to extend and adapt Aggregate Filters to new target classes, other forms of target distortion, and background noise.

1. Scale Invariance

We have demonstrated the Aggregate Filter for simple rotation and need to demonstrate its success with scale invariance. The issue here is the degree to which the target's Fourier spectra will overlap after radical changes.

2. Real Targets

The Aggregate Filter demonstration used an artificial target. Multiple real targets need to be tested to illustrate the effects of unpredictable spectra. These may be in the form of gray scale images or silhouettes.

3. Additive Noise

The Aggregate Filter demonstration did not involve any discussion of additive noise because the effect of any phase-only filter is the same on additive White Noise. One variation on a phase-only filter to obtain some noise rejection is to block out areas of the Fourier plane that have no target energy. This is different from an optimal noise rejection filter where the filter transmittance depends linearly on the expected target signal. The blocked out spectral areas are an appropriate match to the binary phase filter, becoming a binary phase and binary amplitude detection filter.

We recommend determining the optimal multiple target phase-only Aggregate Filter for an additive noise background. The optimal binary amplitude, binary phase target detection filter also needs to be identified.

4. Full Rotation Invariance

We have demonstrated how the phase-only Aggregate Filter improves rotation invariance. Another quantitative comparison is also needed between the Aggregate Filter and the negative effects of attempting rotation invariance by simple rotational averaging (smearing). Smearing reduces the high frequency contribution and is expected to widen the correlation spike and reduce its amplitude.

The test would involve comparing a set of Aggregate Filters designed to detect a 360° target rotation, against a set of rotationally averaged filters for the same target and 360° rotation angle.

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